Freeze-out Conditions from Lattice QCD

Swagato Mukherjee

Physics Department, Brookhaven National Laboratory, Upton, NY 11973, U.S.A.

Abstract

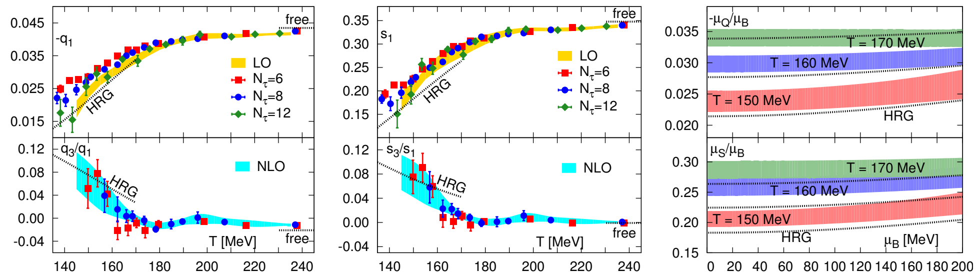
We describe a procedure for determination of freeze-out parameters of heavy-ion collisions through direct comparisons between experimentally measured higher order cumulants of charge fluctuations and first principle (lattice) QCD calculations.

1. Introduction

The QCD critical point is a conjectured second order phase transition point in the temperature (T) and baryon chemical potential (μ_B) plane of the QCD phase diagram. Being a second order phase transition point it will be associated with large correlation lengths, which in turn will be manifested in characteristic large fluctuations. In this vein, one of the major focus in search for the QCD critical point in the Relativistic Heavy-Ion Collider (RHIC)'s Beam Energy Scan (BES) program has been concentrated on measuring higher order cumulants of conserved charge fluctuations, such as net baryon (B), net electric charge (Q) and net strangeness (S).

Experimentally measured hadronic observables in Heavy-Ion Collisions (HIC) characterize the freeze-out condition. The success of statistical hadronization models, based on the thermal hadron spectrum of the Hadrom Resonance Gas (HRG) model, in fitting the experimentally measured hadron yields suggests that freeze-out conditions in HIC can be described by equilibrium thermodynamics characterized by freeze-out temperatures (T^f) and chemical potentials ($\mu_B^f, \mu_Q^f, \mu_S^f$). On the other hand, the QCD critical point will be located along the QCD transition/crossover line in the $T-\mu_B$ plane. Thus in order to observe signatures of the critical point in RHIC BES, the freeze-out of the conserved charge fluctuations must happen at some (T^f, μ_B^f) close to the QCD transition line in the $T-\mu_B$ plane. While for moderate values of μ_B the QCD transition line in the $T-\mu_B$ plane is known from first principle Lattice QCD (LQCD) calculations [1, 2], till now there is no first principle QCD determination of freeze-out parameters associated with the observables related to the conserved charge fluctuations.

Here we outline a procedure for the determination of the freeze-out parameters $(T^f, \mu_B^f, \mu_Q^f, \mu_S^f)$ via direct comparison between first principle LQCD calculations and the experimentally measured cumulants of charge fluctuations. While such a procedure will tell us whether the experimentally measured fluctuations can indeed be described by equilibrium thermodynamics, the freeze-out parameters obtained using this procedure do not necessarily correspond to the chemical freeze-out parameters. Chemical freeze-out indicate irrelevance of inelastic scatterings, but the freeze-out of the conserve charge fluctuations takes place through diffusion processes. It is likely that hadronic final state interactions are important for the freeze-out of charge fluctuations and it is a-priori not clear that the charge fluctuations freeze-out at the chemical freeze-out point. Thus, in order to make ab-initio parameter-free theoretical predictions relevant for the RHIC BES



(a) (b) (c)

Figure 1: (a) Top: Continuum extrapolated LO in μ_B contribution for the electric charge chemical potential as a function of temperature. Bottom: NLO contribution, in units of the LO contribution, as a function of temperature for the electric charge chemical potential. (b) Same as the previous panel, but for the strangeness chemical potential. (c) Electric charge (top) and strangeness (bottom) chemical potential as a function of μ_B for the relevant temperature range $T = 150 - 170$ MeV.

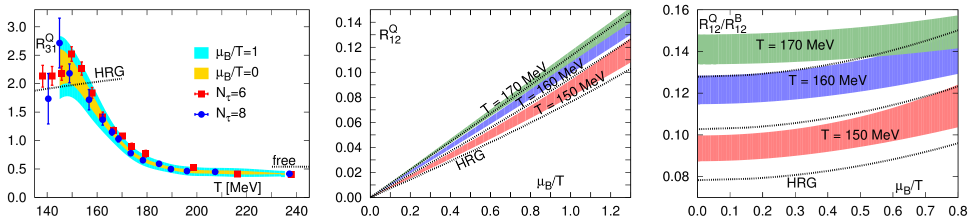
it is crucial to know the freeze-out parameters associated with the conserved charge fluctuations. Once these freeze-out parameters are known from the comparison of the lower order cumulants only then higher order cumulants can be predicted from LQCD calculations along this freeze-out line in a completely parameter-free manner. Subsequent comparisons with the experimental data will clarify to what extent the higher order cumulants contain non-critical/critical signatures.

The LQCD computations that will be presented here were performed using 2+1 flavor Highly Improved Staggered Quarks (HISQ) with a physical strange quark mass and light quark masses corresponding to the Goldstone pion mass of 160 MeV. Three different lattice spacings corresponding to the temporal extents $N_\tau = 6, 8, 12$ were used for the calculations. Further details concerning the LQCD calculations as can be found in Refs. [1, 3]. More detailed discussions regarding the determination of the freeze-out parameters can be found in Ref. [4].

2. Electric charge and strangeness chemical potentials

By using the constraints of initial strangeness neutrality and initial iso-spin asymmetry of the colliding nuclei of HIC the electric charge and strangeness chemical potentials, μ_Q and μ_S respectively, can be treated as dependent parameters of T and μ_B. Assuming spatial homogeneity, the initial strangeness neutrality gives $\langle n_S \rangle = 0$ and the initial iso-spin asymmetry of the colliding nuclei leads to $\langle n_Q \rangle = r \langle n_B \rangle$. Here, n_X denotes the density of the corresponding net conserved charge X and $r = N_p/(N_p + N_n)$ is the ratio of the total number of protons to the total number of protons and neutrons of the initially colliding nuclei. We choose to work with $r = 0.4$, a good approximation for $Au - Au$ as well as $Pb - Pb$ collisions. By making Taylor expansions of $\langle n_X \rangle$ in (μ_B, μ_Q, μ_S) up to $O(\mu_X^3)$ and imposing the above two constraints one can write down the μ_Q and μ_S in terms of the other two independent parameters: $\mu_Q(T, \mu_B) = q_1(T)\mu_B + q_3(T)\mu_B^3$ and $\mu_S(T, \mu_B) = s_1(T)\mu_B + s_3(T)\mu_B^3$.

In Fig. 1(a) we show LQCD results for the Leading Order (LO) contribution $q_1(T)$ (top) and the Next-to-Leading Order (NLO) contribution $q_3(T)$ (bottom) to μ_Q. Similar contributions for the μ_S are shown in Fig. 1(b). These results show that the NLO contributions are less than 10% and well under control for a wide range of the baryon chemical potential $\mu_B \lesssim 200$ MeV, *i.e.* for



(a) (b) (c)

Figure 2: LQCD results for the *thermometer* R^Q_{31} (a) and the *baryometer* R^Q_{12} (b). (c) LQCD results for the double ratio R^Q_{12}/R^B_{12} as a function of μ_B/T.

RHIC energies down to $\sqrt{s_{NN}} \gtrsim 19.6$ GeV. In Fig. 1(c) we show $\mu_Q(T,\mu_B)$ (top) and $\mu_S(T,\mu_B)$ (bottom) as a function of μ_B for the relevant temperature range $T = 150-170$ MeV. Comparisons with the HRG model suggest that the LQCD results differ by 10-15%.

3. Freeze-out temperature and baryon chemical potential

As μ_Q and μ_S are known as a function of T and μ_B all the cumulants of conserved charge fluctuations can be now be expressed only in terms of T and μ_B. Although in RHIC both net proton and net electric charge fluctuations are being measured [5, 6, 7], in LQCD only fluctuations of conserved quantities are accessible. Since the net proton fluctuations may not be quantitatively equal to the net baryon number fluctuations, it is safer to work with the net electric charge fluctuations. We propose to look at two different volume independent ratios formed out of the three lowest order cumulants, mean (M_Q), variance (σ_Q) and skewness (S_Q), of the charge fluctuations—

$$R^Q_{31} \equiv \frac{\chi^Q_3(T,\mu_B)}{\chi^Q_1(T,\mu_B)} = \frac{S_Q\sigma^3_Q}{M_Q} = R^{Q,0}_{31} + R^{Q,2}_{31}\mu^2_B + \cdots \tag{1a}$$

$$R^Q_{12} \equiv \frac{\chi^Q_1(T,\mu_B)}{\chi^Q_2(T,\mu_B)} = \frac{M_Q}{\sigma^2_Q} = R^{Q,1}_{12}\mu_B + R^{Q,3}_{12}\mu^3_B + \cdots\ . \tag{1b}$$

While the cumulants of the net charge distribution M_Q, σ_Q, S_Q are being measured in RHIC BES [5, 6, 7], the generalized higher order charge susceptibilities $\chi^Q_n(T,\mu_B) = \partial^n \ln \mathcal{Z}/\partial(\mu_B/T)^n$ can be calculated using LQCD via Taylor expansions $\chi^Q_n(T,\mu_B) = \sum_{k=0}\chi^Q_{n+k}(T,0)(\mu_B/T)^n/k!$. In a similar manner we can Taylor expand the ratios R^Q_{31} and R^Q_{12} themselves up to NLO in μ_B as shown in Eq. (1a) and Eq. (1b) respectively. Note that, in the LO the ratio R^Q_{31} is independent of μ_B but the LO term for the ratio R^Q_{12} is proportional to μ_B. Thus, R^Q_{31} can be used as the *thermometer* to determine T^f and R^Q_{12} can used as the *baryometer* to fix μ^f_B.

In Fig. 2(a) we show the LQCD results for the ratio R^Q_{31}. The estimated NLO correction to this ratio is at most 10% for $\mu_B/T \approx 1$ over the whole T range. The broader band in the figure depicts the range of R^Q_{31} for $\mu_B/T = 1$ including the NLO correction and the thinner band depicts the LO results, *i.e.* for $\mu_B = 0$. The LQCD calculations for the *thermometer* R^Q_{31} shows a characteristic

T dependence and large deviations from the HRG results within the relevant temperature range $T = 150 - 170$ MeV. For example, by comparing with these QCD results an experimentally measured value of $R^2_{31} \gtrsim 2$ will indicate a freeze-out temperature $T^f \lesssim 150$ MeV, $R^Q_{31} \approx 1.5$ will give $T^f \approx 160$ MeV and $R^Q_{31} \lesssim 1$ will mean $T^f \gtrsim 170$ MeV.

After fixing T^f from the *thermometer* R^Q_{31} we can use the *baryometer* R^Q_{12} to determine the freeze-out baryon chemical potential μ^f_B. Fig. 2(b) shows LQCD results for the ratio R^Q_{12} as a function of μ_B in the relevant temperatures interval $T = 150 - 170$ MeV using continuum extrapolated LO results and adding contributions up to NLO in μ_B. In this temperature range the NLO corrections are well under control, less than 10%, for $\mu_B \lesssim 200$ MeV, *i.e.* for RHIC BES energies of $\sqrt{s_{NN}} \gtrsim 19.6$ GeV. By comparing the experimentally measured values of R^Q_{12} with these first principle QCD calculations one can determine μ^f_B. As a concrete example, choosing $T^f = 160$ MeV, an experimental value of $R^Q_{12} = 0.01 - 0.02$ will give $\mu^f_B/T^f = 0.1 - 0.2$, $R^Q_{12} = 0.03 - 0.04$ will suggest $\mu^f_B/T^f = 0.3 - 0.4$ and $R^Q_{12} = 0.05 - 0.08$ will indicate $\mu^f_B/T^f = 0.5 - 0.8$.

4. Thermodynamic consistency

Such a determination of the freeze-out parameters for a given beam energy will help us in understanding whether at that beam energy higher order cumulants of conserved charge fluctuations can be consistently described within the framework of equilibrium thermodynamics. If the experimentally measured fluctuations are indeed described by equilibrium thermodynamics characterized by unique values of temperature and chemical potential then other volume independent ratios of cumulants must also have unique values as predicted by (L)QCD calculations at the same (T^f, μ^f_B). As an example. in Fig. 2(c) we show the LQCD predictions for the double ratio R^Q_{12}/R^B_{12} as a function of μ_B/T. For a pre-determined values of the freeze-out parameters T^f and μ^f_B this ratio has a unique value consistent with equilibrium thermodynamics. If the experimentally measured observables contain the correct physics of equilibrium thermodynamic fluctuations then they must agree with this (L)QCD prediction.

Acknowledgments

The author is supported by contract DE-AC02-98CH10886 with the U.S. Department of Energy.

References

References

[1] A. Bazavov *et al.* [HotQCD Collaboration], Phys. Rev. D **85**, 054503 (2012).
[2] O. Kaczmarek *et al.*, Phys. Rev. D **83**, 014504 (2011).
[3] A. Bazavov *et al.* [HotQCD Collaboration], Phys. Rev. D **86**, 034509 (2012).
[4] A. Bazavov *et al.*, arXiv:1208.1220 [hep-lat] (to appear in Phys. Rev. Lett.)
[5] X. Luo [STAR Collaboration], this proceedings.
[6] D. McDonald [STAR Collaboration], this proceedings.
[7] J. Mitchell [PHENIX Collaboration], this proceedings.